term 1

1.) The effective annual rate of discount has been 7% for the last 10 years. Prior to that, it was 9%. A bank account has a balance of $800 today. A single deposit of $X was placed in the account 17 years ago. Calculate X.

2.) Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns an annual effective discount rate of d.

The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X.

Calculate X.

3.) You are given an annuity-immediate with 11 annual payments of $100 and a final payment of X at the end of 12 years. At an annual effective interest rate of 3.5%, the present value at time 0 of all payments is $1000. Using an annual effective interest rate of 1%, calculate the present value at the beginning of the ninth year of all remaining payments.

4.) At a force of interest $\delta_t = \frac{0.1}{1+0.1t}$, the following payments have the same present value:

(i) X at the end of year 5 plus 2X at the end of year 10; and
(ii) Y at the end of year 14.

Calculate X ÷ Y.

20 088.14

5.) Wayne is saving for his retirement. Wayne turned 15 years old today and will make deposits of $300 dollars at the beginning of each year for the next 12 years. Just before Wayne turns 27 years old he decides to increase his next payment to $2500. He continues making $2500 annual payments until his last payment at age 64. His bank account earns 6.5% interest effective annual.

606.71

The day Wayne turns age 65 he would like to begin receiving level payments from his bank account. What level payment amount will he receive each year for the next 20 years?

6.) The following three series of payments have the same present value of P:

(i) A perpetuity-immediate of 5 per year at an annual effective interest rate i
(ii) A 10-year annuity-immediate of X per year at an annual effective interest rate of 3i
(iii) A 10-year annuity-due of .95694X per year at an annual effective interest rate of 3i.

Calculate P.

7.) A bank agrees to lend you \$10,000 now and X three years later in exchange for a single repayment of \$75,000 at the end of 10 years.

The bank charges interest at an annual effective rate of 6% for the first 5 years and at a force of interest $\delta_t = \frac{1}{1+t}$ for $t \geq 5$.

Determine X.

8.) Tim can receive one of the following two payment streams:

(i) \$150 at time 0, \$460 at time 7, and \$580 at time 14
(ii) \$1705 at time 14.

At an annual effective interest rate of i, the present values of the two streams are equal.

Calculate i.

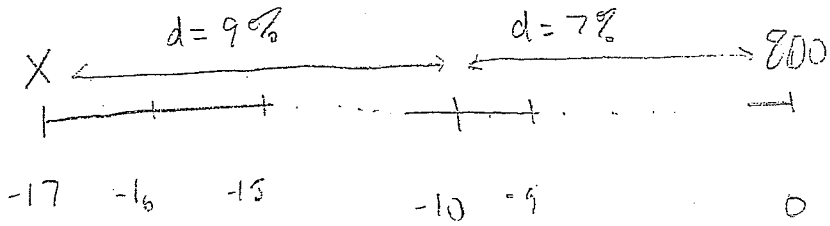
9.) A perpetuity-immediate pays *X* per year. Brian receives the first *n* payments, Colleen receives the next *n* payments, and Jeff receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and Jeff's share is *K*.

Calculate *K*.

10.) Betty receives payments of \$100 at the beginning of each year, including today, forever. Smitty receives payments of \$X at the end of each year, starting 5 years from today (ie. first payment at t=5), forever. The present values of their payments are the same at a constant force of interest equal to 7.5%. Calculate X.

Solutions to Term Test #1

(1)



$$X \cdot \left[\frac{1}{(1-.09)^7}\right] \cdot \left[\frac{1}{(1-.07)^{10}}\right] = 800$$

$$\Rightarrow \boxed{X = \$200.08}$$

(2) Bruce

FV at $t=10 \Rightarrow 100(1+i)^{10}$

interest earned btw $t=10$ and $t=11 \Rightarrow 100(1+i)^{10} \cdot i$

Robbie

interest earned in 17th year $\Rightarrow 50(1+i)^{16} \cdot i$

$$X = 100(1+i)^{10} \cdot i = 50(1+i)^{16} \cdot i \rightarrow \cancel{100}^{2}(1+i)^{10} \cdot \cancel{i} = \cancel{50}^{1}(1+i)^{16} \cdot \cancel{i} \Rightarrow$$

$$2 = \frac{(1+i)^{16}}{(1+i)^{10}} \Rightarrow i = 12.2462\% \qquad d = \frac{i}{1+i} \qquad d = 10.91\%$$

$$X = 100(1.122462)^{10} \cdot (.122462) = \boxed{38.88}$$

(3)

$$1000 = 100\, a_{\overline{11}|3.5\%} + X v^{12}$$

$$1000 = 100\,(9.00155) + X\cdot(.66178)$$

$$X = 150.87$$

PV of remaining pymts at 1% =>

PV?	100	100	100	150.87
8	9	10	11	12

$$100\, a_{\overline{3}|1\%} + 150.87\, v^{4}_{1\%} = \boxed{439.08}$$

(4) $e^{\int_0^t \delta_t\, dt} = e^{\int_0^t \frac{0.1}{1+0.1t}\, dt} = e^{\left[\ln(1+0.1t)\big|_0^t\right]} = \dfrac{1+0.1t}{1+0}$

(i) $\dfrac{X}{1+0.1(5)} + \dfrac{2X}{1+0.1(10)}$ (ii) $\dfrac{Y}{1+0.1(14)}$ $\Rightarrow \dfrac{X}{1.5} + \dfrac{2X}{2} = \dfrac{Y}{2.4}$

$4X = Y$ $\boxed{\dfrac{X}{Y} = \dfrac{1}{4}}$

(5) $$300\,\ddot{s}_{\overline{12}|}\cdot(1+i)^{38} + 2500\,\ddot{s}_{\overline{38}|} = X\cdot \ddot{a}_{\overline{20}|}$$

$$60753.81 + 407434.08 = X\cdot \boxed{11.01851} \rightarrow 11.73471$$

$\therefore$ $\boxed{X = \$42491}$

$\boxed{39\,897.70}$

6\. (i) $P = \frac{5}{i}$ (ii) $P = X \cdot a_{\overline{10}|3i}$ (iii) $(.95694) \cdot X \cdot \ddot{a}_{\overline{10}|3i}$

looking at (ii) and (iii):

$$X \cdot a_{\overline{10}|3i} = (.95694) \cdot X \cdot \ddot{a}_{\overline{10}|3i} \Rightarrow \cancel{X} \frac{\cancel{(1-v^{10})}}{\cancel{3i}} = (.95694) \cdot \cancel{X} \cdot \frac{\cancel{(1-v^{10})}(1\cdot}{\cancel{3i}}$$

$$\Rightarrow 1 = (1+3i) \cdot (.95694) \qquad (1+3i) = \frac{1}{.95694} \qquad 3i = .044998$$

$$\Rightarrow i = 1.5\% \qquad \Rightarrow P = \frac{5}{i} \qquad P = \frac{5}{.015} \qquad \boxed{P = 333.35}$$

7\.

$$10\,000 + \frac{X}{(1+i)^3} = 75000 \cdot \left[\exp\left[-\int_5^{10} \frac{1}{1+t}\,dt\right]\right] \cdot \left(\frac{1}{(1+i)^5}\right)$$

$$\exp\left[-\int_5^{10} \frac{1}{1+t}\,dt\right] = \exp\left[-\ln(1+t)\Big|_5^{10}\right] = e^{[\ln(6) - \ln(11)]} = \frac{6}{11}$$

$$\Rightarrow 10000 + (.83962)X = 75000\left(\frac{6}{11}\right) \cdot (.74726)$$

$$X = \frac{30569.65 - 10000}{.83962}$$

$$\boxed{X = {}^{\#}24\,499}$$

8. If PV of (i) and (ii) is equal than FV_{14} must be equal as well.

$\therefore \quad 150(1+i)^{14} + 460(1+i)^7 + 580 = 1705$ let $x = (1+i)^7$

$150x^2 + 460x - 1125 = 0$ use quadratic to get x

$x = 1.60531$ or $\cancel{x = -4.67198}$ $\quad (1+i)^7 = 1.60531 \quad \boxed{i = 7\%}$

9. Brian: $.40\, a_{\overline{\infty}|} = a_{\overline{n}|}$ Colleen: $PV = v^n \cdot a_{\overline{n}|}$

$$\frac{.40}{i} = \frac{1-v^n}{i}$$

$$PV = \frac{(.6)(1-.6)}{i} = \frac{.24}{i}$$

$v^n = .60$

Jeff: $PV = v^{2n} \cdot a_{\overline{\infty}|} = \frac{(.6)^2}{i} = \frac{.36}{i}$

$\therefore \quad a_{\overline{\infty}|} = \frac{.40}{i} + \frac{.24}{i} + \frac{.36}{i} \Rightarrow a_{\overline{\infty}|} = (.40)\, a_{\overline{\infty}|} + (.24)\, a_{\overline{\infty}|} + \underline{\underline{(.36)\, a_{\overline{\infty}|}}}$

$\boxed{\therefore K = 36\%}$

10. Betty $PV = \frac{100}{i} \cdot (1+i) = \frac{100}{d}$ $\quad e^{\delta} = 1+i$

$i = 7.788\%$

Smitty $PV = \frac{X}{i} \cdot v^4$

$1383.96 = X \cdot 9.511797$ $\quad \boxed{X = \$145.50}$